Analysis of Small Dilation Detection in Coronary Angiography

T. B. Smith[1], and K. S. Nayak[1]
[1]Electrical Engineering, University of Southern California, Los Angeles, CA, United States

Introduction: MRI may provide a non-invasive detection mechanism for very early stages of heart disease by assessing the coronary arterial dilation response to endothelial-dependent stimuli, which is correlated with coronary health [1,2]. In these early studies, cross-sectional images of the coronary lumen acquired before and after administration of such stimuli are fit to circular templates to estimate the change in lumen area. Mean lumen area changes of about –10% (constriction) to 5% (dilation) have been reported [1,2]. In this work, we develop a statistical framework for the detection of subtle changes in lumen area from two images. We use this framework to relate detection performance to SNR requirements and minimum detectable dilation.

Methods: Our analysis is based on lumen area change detection in the difference image, rather than separately estimating the area in each image. We consider an ideal case in which the two images are perfectly registered and the lumen cross sections are exactly circular and free of off resonance artifacts, so that the difference image contains only noise and an annulus created from the change in area. The image-domain signal from lumen with diameter W is modeled as a circularly symmetric function, rect(r/W). When the lumen diameters are W and W+d, with $0<d<<W$, the difference signal at k-space location $\rho = (k_x^2 + k_y^2)^{0.5}$ is approximately $s(\rho) \approx 0.5\pi dWJ_0(\pi\rho W)$, where J_0 is the 0^{th} order Bessel function of the 1^{st} kind. Fig. 1 shows an example simulated vessel.

The measured difference signal is $\mathbf{y} = \mathbf{s} + \mathbf{n}$, where $\mathbf{s}$ is the sampled signal $s(\rho)$ and $\mathbf{n}$ is $N(\mathbf{0},2\sigma^2 I)$. Each image of the vessel lumen therefore has SNR $1/\sigma$. Note that our model assumes no specific k-space trajectory or resampling. Detection of the unknown, deterministic dilation is a Neyman-Pearson hypothesis test with H_0 = {no change, $\mathbf{y} = \mathbf{n}$} and H_1 = {change in area, $\mathbf{y} = \mathbf{s} + \mathbf{n}$}, and the maximum likelihood (ML) decision rule is the correlation detector, or matched filter (MF), in which we reject H_0 if $\mathbf{s}^T\mathbf{y} > \gamma$ and accept H_0 otherwise [3]. The threshold γ is chosen for a desired probability of false alarm, P_{FA}, from $\gamma = \sqrt{2}\|\mathbf{s}\|\sigma Q^{-1}(P_{FA})$, where Q(x) is the tail probability of an $N(0,1)$ distribution. We can also define the probability of detection, P_D, as $\sqrt{2}\|\mathbf{s}\|\sigma Q^{-1}(P_D) = \gamma - \|\mathbf{s}\|^2$. Thus, detection performance and lumen image SNR are related by $Q^{-1}(P_{FA}) - Q^{-1}(P_D) = \|\mathbf{s}\|\cdot SNR/\sqrt{2}$. Here, $\|\mathbf{s}\|$ captures the dependence on the diameters W and W+d. For a Cartesian sampling pattern, $\|\mathbf{s}\|^2 \approx 0.866Wd^2/\delta^3$, where δ is the image resolution. Thus, detection performance improves linearly with d, and with $\sqrt{W}$. Note also that the difference image SNR is only $d/(\sqrt{8}\delta\sigma)$.

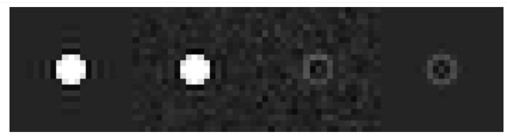

Fig. 1: From left to right: simulated lumen, lumen with noise, difference image with noise $\mathbf{s} + \mathbf{n}$, and MF $\hat{\mathbf{s}}$. Lumen SNR = 75, W = 3.1 mm, 7% dilation, and δ = 0.7 mm. High SNR is used to make $\mathbf{s}$ apparent in noise.

Because $\mathbf{s}$ contains unknown parameters W and d, we perform MF with $\hat{\mathbf{s}}$ instead, in which W and d have been replaced by their ML estimates, using a generalized likelihood ratio test (GLRT). Due to the nonlinear dependence on W, the ML parameter estimates are found by minimizing $\|\mathbf{y}-\hat{\mathbf{s}}\|^2$ using grid search [3].

Results and Discussion: To examine how detection performance relates to dilation and SNR, we assume the following scan parameters (unless otherwise stated): image SNR = 20, δ = 0.7 mm, W = 3.1 mm [4], W+d = 3.21 mm (7% area change), and Cartesian sampling. Fig. 2 shows the receiver operating characteristic (ROC) behavior assuming no estimation error ($\hat{\mathbf{s}} = \mathbf{s}$) for various dilations. Fig. 3 is another ROC showing the minimum required image SNR. Even under these ideal conditions, the minimum SNR necessary for robust detection is quite large considering the fine image resolution. Here, the observable $\mathbf{y}$ (the difference image) has 18 times smaller SNR than the lumen images. A Monte Carlo simulation was performed to compare detection with no estimation error (MF) and the GLRT. The ROC curves, shown in Fig. 4, show the impact of estimator variance. This simulated GLRT ROC is a more realistic depiction of true detection performance than those in Figs. 2 and 3.

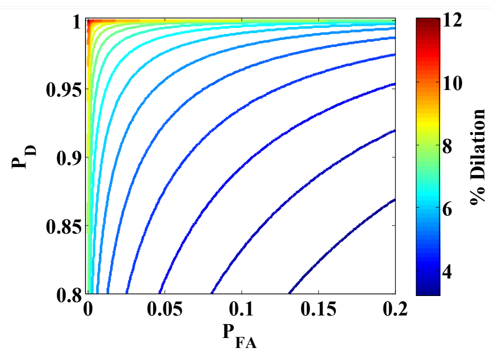

Fig. 2: ROC (with no diameter estimation error) showing the min. detectable dilation for SNR = 20.

This framework enables a quantitative analysis of the detection of subtle lumen dilations. Future work will incorporate non-idealities such as imperfect fat suppression and lumen registration, and characterize the ML estimators.

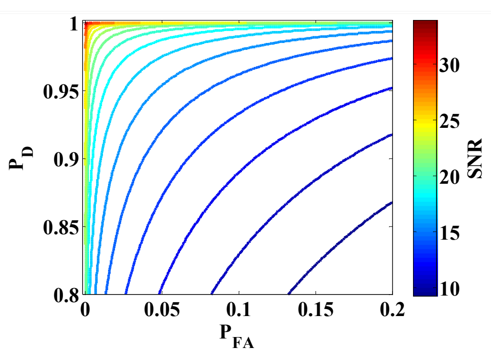

Fig. 3: ROC (with no diameter estimation error) showing the min. SNR for detection of 7% dilation.

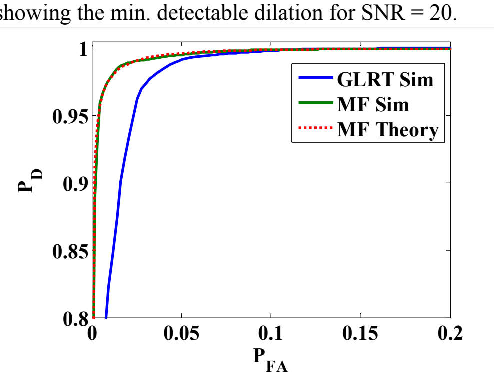

Fig. 4: ROC for GLRT and MF simulations. Estimation errors in the GLRT degrade detection performance.

References: [1] Jain, et al. Circulation 2006:114(II):541. [2] Maxwell, et al. Card. Drugs and Therapy 2000;14(3): 309-316. [3] Kay. Fundamentals of Stat. Signal Proc., Englewood Cliffs, NJ: Pren.-Hall, 1998. [4] Haskell, et al. Circulation 1993;87:1076-1082.