Graphical Derivation of the Steady-State Magnetization in Balanced SSFP MRI

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Introduction: Balanced steady-state free precession (SSFP) imaging provides exceptionally high signal-to-noise ratio (SNR) efficiency and useful T_2/T_1 based contrast [1]. To analyze the SSFP signal, matrix calculations have been traditionally used. Dharmakumar et al. recently suggested a geometry-based description of SSFP by identifying the steady-state locus of magnetization at the time immediately after each excitation pulse [2]. We introduce a new geometry-based derivation of the steady-state signal which yields a simple explanation of how the steady-state signal is affected by variation of imaging and object parameters, and matches known signal profiles. We introduce a new parameter "effective flip angle", which along with T_2/T_1, uniquely determines steady-state signal, and which combines the effects of imaging flip angle and off-resonance precession.

Balanced SSFP on-resonance: For alternating balanced SSFP (α,-α) exactly on-resonance (Δf=0), the magnetizations in the steady state are shown in Fig. 1 (relaxation is exaggerated). The four magnetizations all have exactly the same magnitude; that is, $|M(0)| = |M(TR)|$. Consequently, the net relaxation vector over a TR (ΔM_{XY}, ΔM_Z) must be perpendicular to the magnetization vector (M_{XY}, M_Z). This yields the following relationship between M_{XY} and M_Z.

$$M_{XY}^{\ 2} \cdot \frac{T_1}{T_2} + \left(M_Z - \frac{M_0}{2}\right)^2 = \left(\frac{M_0}{2}\right)^2 \qquad \text{(Eq. 1)}$$

which describes an ellipse. The trajectory of (M_{XY}, M_Z) in the steady state is shown as a function of α in Fig. 2. This ellipse pattern was shown by Hennig et al. [3] in a description of the transition to steady state, and now has an intuitive explanation. From Fig. 2, one can see that the greatest possible transverse signal amplitude $M_0/2 \cdot \sqrt{T_2/T_1}$ is achieved when (M_{XY}, M_Z) reaches the rightmost point, at which $\tan(\alpha/2) = \sqrt{T_2/T_1}$ or $\cos\alpha = (T_1-T_2)/(T_1+T_2)$. The maximum transverse signal for balanced SSFP confirms T_2/T_1 contrast.

Balanced SSFP with off-resonance: To introduce off-resonance to the graphical analysis, let the dephasing angle within TR be θ. Figure 3a shows the path that SSFP magnetization follows in the steady state. Magnetizations are refocused at TE=TR/2 regardless of resonance offset [4]. As shown in Fig. 3b, the two magnetizations at the echo form an angle α' which is equivalent to the sum of two angles between the z-axis and each magnetization before and after the RF tip in Fig. 3a (assuming that the effect of relaxation is negligible during TE). Balanced SSFP with off-resonance can now be modeled as a balanced SSFP on-resonance but with effective flip angle α'. It can be geometrically shown that $\tan(\alpha'/2) = \tan(\alpha/2)/\cos(\theta/2)$ (Eq. 2). Therefore, the balanced SSFP magnetization at TR/2 with off-resonance will still lie along the same curve shown in Fig. 2, but with effective flip angle α'. Equation 2 shows that the effective flip angle α' is a function of α and off-resonance angle θ. On resonance, α' is the same as α, and as θ increases from 0° to 180°, α' also increases and reaches 180°. Figure 4 illustrates how the magnetization and steady-state signal vary according to flip angle α. Since α' remains around its initial value α on-resonance and then increases towards 180° as θ increases, the corresponding magnetization along the Mxy-Mz trajectory moves slowly in the upper portion of semi-ellipse, and faster when going through the lower portion.

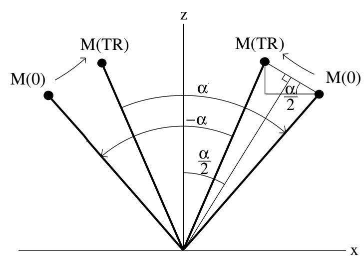

***Fig. 1:** Steady-state magnetizations of alternating balanced SSFP on resonance with exaggerated relaxation.*

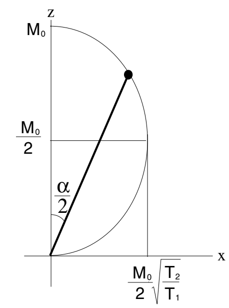

***Fig. 2:** Magnetization (M_{XY}, M_Z) in the steady state as a function of α.*

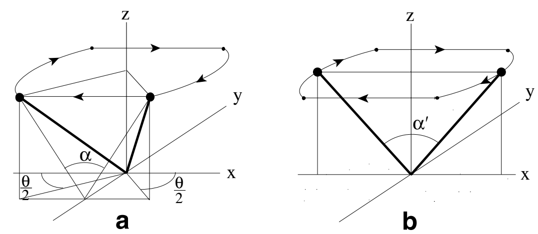

***Fig. 3:** Balanced SSFP with off-resonance precession in the steady state. **a**: Magnetizations immediately before and after RF pulse. **b**: Magnetizations at TE=TR/2.*

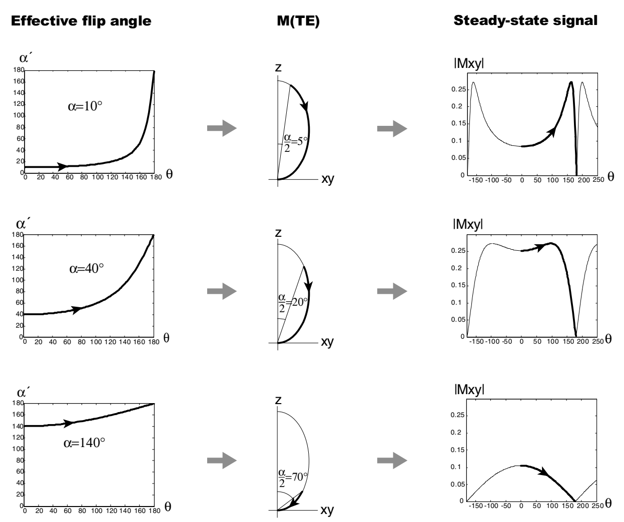

***Fig. 4:** Effective flip angle, magnetization at echo time, and steady-state signal as a function of prescribed flip angle and off-resonance θ. α=10° (top row), α=40° (middle row), and α=140° (bottom row), T_1=1000ms, T_2=300ms, and TE=TR/2.*

Discussion: The balanced SSFP signal profile has been derived in a graphical manner which gives a new intuition for SSFP signal variations. We have shown that, in the steady state, T_1 and T_2 relaxation plays the critical role in determining the signal profile, where the net relaxation vector is perpendicular to the magnetization vector. This explains the phenomenon of magnetization in the steady state always falling on an ellipse. We also present a new perspective where off-resonance can be understood as on-resonance with a different effective flip angle. This graphical analysis could be expanded to more complex SSFP-like sequences or to develop tailored excitations to maximize uniformity of the SSFP signal.

References

[1] Carr H., Phys Rev, 112:1693, 1958.
[2] Dharmakumar R., et al., Concepts in MR Part A, 26: 1, 2005.
[3] Hennig J., et al., MRM, 48:801, 2002.
[4] Scheffler K., et al., MRM, 49:395, 2003.